

FACULTAD DE CIENCIA Y TECNOLOGÍA

STAFF WEEK

# LINEAR ALGEBRA IN ISABELLE /HOL

15-17 NOVEMBER 2017

Complejo Científico Tecnológico  
Seminario Mirian Andrés



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## ORGANISERS

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## PROGRAMME

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Wednesday, 15 November 2017

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11.00 horas

### The Perron–Frobenius Theorem in Isabelle/HOL–Transferring between Matrix–Representations

RENÉ THIEMANN  
University of Innsbruck

Matrix interpretations are widely used in automated complexity analysis. For these interpretations, certification boils down to determining the growth rate of  $A^n$  for a fixed non-negative real matrix  $A$ .

Since the direct approach to compute the growth rate of  $A$  via algebraic numbers is quite slow, in this work we formalize the Perron–Frobenius theorem: it permits us to avoid algebraic numbers, so that our new certification algorithm only has to perform simple arithmetic operations. To cover the theorem in its full extend, we further establish a connection between two different Isabelle/HOL libraries on matrices, which enables an easy exchange of theorems between both libraries.

14.00 horas

### Rigorous Numerics and Linear Algebra in Isabelle/HOL

FABIAN IMMLER  
Technische Universität München

In this talk, I will present which concepts and results from linear algebra are used in my verified rigorous numerical ODE solver. Instead of matrices I work with a type of bounded linear functions. Matrix operations are represented as (lists of) deeply embedded arithmetic expressions. I set up Lifting and Transfer to convert between the type class of Euclidean space and vectors or matrices.

Thursday, 16 November 2017

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9.00 horas

### Recent development in Lean and its analysis

JOHANNES HÖLZL  
Vrije Universiteit Amsterdam

I will present the current state of analysis in Lean 3. The formalization of the reals is based on the completion of uniform spaces, requiring a more extensive filter library. While the analysis in Lean is developed up to the Lebesgue measure, I mostly want to focus on topological spaces and measurable spaces, and compare their formalization with the one in Isabelle.

11.00 horas

### Semi–Automatic Asymptotics in Isabelle/HOL

MANUEL EBERL  
Technische Universität München

Computer Algebra Systems can easily compute limits and asymptotic expansions of complicated real functions; interactive theorem provers, on the other hand, provide very little support for such problems and proving asymptotic properties of a function often involves long and tedious manual proofs. In this talk, I will present my work about bringing automation for real-valued asymptotics to Isabelle/HOL using multi-series expansions. This yields a procedure to automatically prove limits and 'Big-O' estimates of real-valued functions similarly to computer algebra systems like Mathematica and Maple – but while proving every step of the process correct.

Friday, 17 November 2017

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11.00 horas

### An Isabelle/HOL formalisation of Green's Theorem

MOHAMMAD ABDULAZIZ  
Technische Universität München

We describe a formalisation of Green's theorem, a fundamental result of multivariate calculus, in Isabelle/HOL. The theorem statement that we formalise is enough for most applications, especially in physics and engineering. An interesting aspect of our proof is that we neither formalise orientations nor region boundaries explicitly, with respect to the outwards-pointing normal vector. Instead we refer to homological equivalences between paths.